Kochen-Specker: Beyond Quantum Theory?

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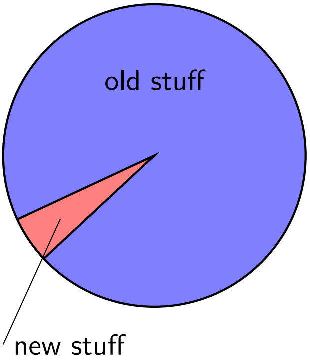
Menu



1. What does the Kochen-Specker theorem state?
2. “Nullifying” the theorem: the finite precision argument
3. Reconsidering non-contextuality
4. Beyond quantum theory?

What is the Kochen-Specker theorem?

In conversation: There is no 2-valued homomorphism on the lattice of projection operators of a Hilbert space of dimension greater than 2.

Stanford encyclopedia: The theorem demonstrates the impossibility of a certain type of interpretation of QM in terms of hidden variables.

Wikipedia: The impossibility that quantum mechanical observables represent “elements of physical reality”. More specifically, the theorem excludes hidden variable theories that require *elements of physical reality to be non-contextual* (i.e. independent of the measurement arrangement).

The theorem

Theorem

Let $\mathcal{H}$ be a Hilbert space with $\dim(\mathcal{H}) > 2$ and $\mathcal{O}_{\rm sa}$ the set of self-adjoint operators. Then there is no function $\lambda : \mathcal{O}_{\rm sa} \to \mathbb{R}$ such that

$$\begin{aligned} \lambda(A) &\in \sigma(A), \\ \lambda(f(A)) &= f(\lambda(A)), \end{aligned} \tag{1}$$

for every $A \in \mathcal{O}_{\rm sa}$, and real-valued Borel functions f.

Why care? The existence of λ is a "natural" requirement for any hidden variable interpretation. Three types of assumptions:

1. Hidden variables
2. Constraints from quantum mechanics
3. Mathematical niceties

$$\mathrm{HV} \wedge \mathrm{QM}_{\mathrm{KS}} \wedge \mathrm{MN} \to \bot \tag{2}$$

The assumptions

Hidden variables

- VD Every observable $\mathcal{A} \in \mathcal{Obs}$ has a unique definite value at all times.
- FM A measurement of $\mathcal{A}$ reveals the value of $\mathcal{A}$ just before the measurement.

Constraints from QM

- OP Every observable $\mathcal{A} \in \mathcal{Obs}$ is associated with an operator $A \in \mathcal{O}_{\mathrm{sa}}$.
- VP A measurement of $\mathcal{A}$ reveals a value a in $\sigma(A)$.
- WCoP If $\mathcal{A}_1, \mathcal{A}_2$ correspond to commuting operators A_1, A_2, then $\exists$ comeasurable observables $\mathcal{A}_1', \mathcal{A}_2'$ corresponding to A_1, A_2.
- EFR If $\mathcal{A}_1, \mathcal{A}_2$ are comeasurable and $A_2 = f(A_1)$, then $a_2 = f(a_1)$.

Mathematical niceties

- NC Every $A \in \mathcal{O}_{\mathrm{sa}}$ is associated with at most one $\mathcal{A} \in \mathcal{Obs}$.
- IP Every $A \in \mathcal{O}_{\mathrm{sa}}$ is associated with at least one $\mathcal{A} \in \mathcal{Obs}$.

“Nullifying” the theorem: The finite precision argument

Defending IP

For some observables [sic], in fact for the majority of them (such as xyp_z), nobody seriously believes that a measuring apparatus exists. – Wigner 1963

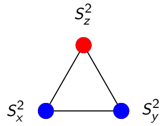
Spin measurements on a spin-1 particle can be measured

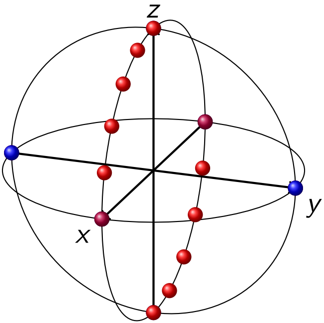
$$\begin{gathered}\left[S_{r_1}^2, S_{r_2}^2\right] = 0 \text{ iff } r_1 \perp r_2, \\ S_x^2 + S_y^2 + S_z^2 = 2.\end{gathered} \tag{3}$$

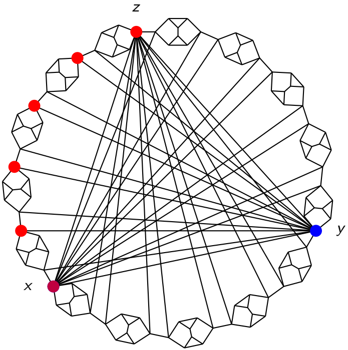
$\mathrm{IP} \mapsto \mathrm{IP}_{\mathrm{KS}}$: There is a finite set D_{KS} such that S_r^2 corresponds to an observable for every $r \in D_{\mathrm{KS}}$.

Rejecting IP/Coloring $\mathbb{S}^2$

$$(S_x^2, S_y^2, S_z^2) \in \{(0,1,1),(1,0,1),(1,1,0)\}$$







Meyer 1999: $\mathbb{S}^2 \cap \mathbb{Q}^3$ is colorable.

Clifton & Kent 2000: Born rule can be recovered.

Reconsidering non-contextuality

What is contextuality?

It was tacitly assumed that measurement of an observable must yield the same value independently of what other measurements may be made simultaneously. – Bell 1966

Suppose

$$[A, B] = [A, C] = 0 \neq [B, C] \tag{4}$$

and joint measurement of (A, B) yields value a for A, then

if instead (A, C) had been measured,

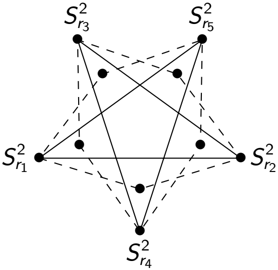
then the outcome would also be a.

$\neg\mathrm{NC}$ There are multiple observable corresponding to A, one for each context.

$\neg\mathrm{VD}$ A has not a *unique* definite value, but one for every context.

$\neg\mathrm{IP}$ A has the same value in every context, but can be measured in at most one context.

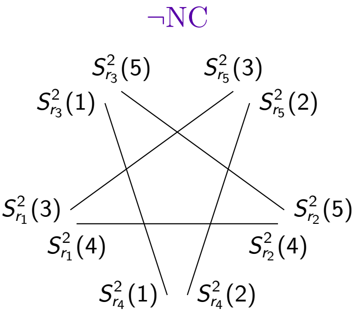
The Klyachko inequality

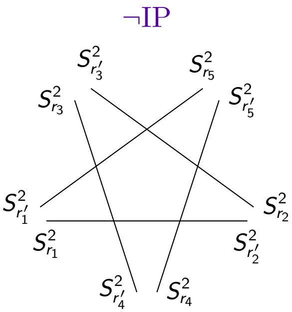


$S^2_{r_1}$	$S^2_{r_2}$	$S^2_{r_3}$	$S^2_{r_4}$	$S^2_{r_5}$	Sum
1	1	1	1	1	5
0	1	1	1	1	4
⋮	⋮	⋮	⋮	⋮	⋮
1	1	1	1	0	4
0	1	0	1	1	3
⋮	⋮	⋮	⋮	⋮	⋮
1	1	0	1	0	3

$$\mathbb{E}\left(S^2_{r_1}\right) + \mathbb{E}\left(S^2_{r_2}\right) + \mathbb{E}\left(S^2_{r_3}\right) + \mathbb{E}\left(S^2_{r_4}\right) + \mathbb{E}\left(S^2_{r_5}\right) \geq 3 \qquad (5)$$

Violating the Klyachko inequality





Measurement non-contextuality: if

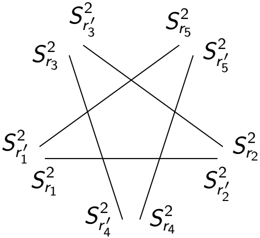
$$\mathbb{P}[A_1 = a|P] = \mathbb{P}[A_2 = a|P]$$

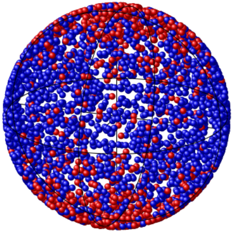
for all outcomes a and preparations P, then $A_1 = A_2$.

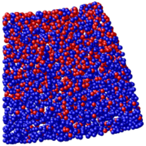
- Quantification over P assumes QM.
- Equality of probabilities cannot be verified.

Beyond quantum theory?

What do violations show?







> *Contextuality is not the essential issue. Instead, the Bell-KS theorem establishes the existence of a fundamental limitation on our ability to determine the values of the hidden variables by measurement. – Appleby 2004*

But how to make this idea precise...