How ψ-ontic are ψ-ontic models? A case study

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Rose is a Rose is a Rose

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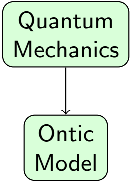
A student bursts into the study of his professor and calls out: "Dear professor, dear professor! I have discovered a perpetual motion of the second kind!" The professor scarcely takes his eyes of his book and curtly replies: "Come back when you have found a neighborhood U of a state x_0 of such a kind that every $x \in U$ is connected with x_0 by an adiabat." – Walter

Outline

1. Ontic models framework.
2. When is an ontic model ψ-ontic?
3. The Kochen-Specker Theorem.
4. The ontic models of Meyer, Clifton and Kent (MKC).
5. The MKC models are ψ-ontic.
6. The MKC models are not very ψ-ontic.

Ontic models as complete descriptions



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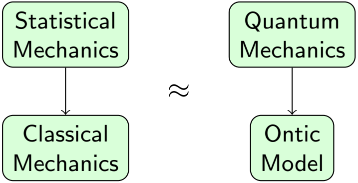

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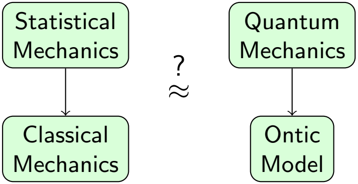

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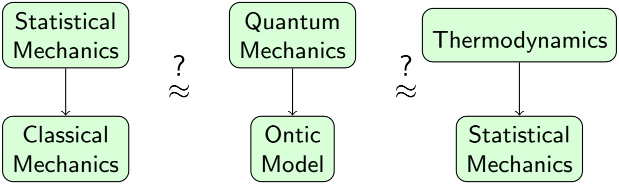

- Do all phenomena described by SM also have a purely classical mechanical description?
- Theorems that constraint ontic models characterize the possible reducing theories for quantum mechanics.

Operational Models and Ontic Models

Operational model $(\mathcal{P}, \mathcal{M})$

$P \in \mathcal{P}$ is a preparation, $M \in \mathcal{M}$ is a measurement,

$\mathbb{P}(m|M, P)$ probability of outcome m for measurement M performed on a system prepared according to P.

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Quantum mechanics as an operational model:

- $\mathcal{P}$ Set of quantum states,
- $\mathcal{M}$ Set of self-adjoint operators,

$$\mathbb{P}(a|A, |\psi\rangle) = \left\langle \psi \middle| P_a^A \middle| \psi \right\rangle .$$

Operational Models and Ontic Models

Ontic model (Λ, Π, Ξ)

$\lambda \in \Lambda$	is an ontic state,	$\xi \in \Xi$	is a Markov kernel
$\mu \in \Pi$	is a probability measure,		from Λ to Ω_ξ.

$\forall (P, M)$ there exists (μ_P, ξ_M) such that

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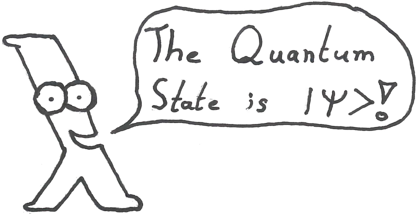
Theorem: An ontic model always exists

"Proof":

$$\Lambda = \mathcal{P},\ \mu_P(\{P'\}) = \delta_{PP'},\ \xi_M(m|P) = \mathbb{P}(m|M, P).$$

ψ-ontic Ontic Models

The ontic state tells us what the quantum state is



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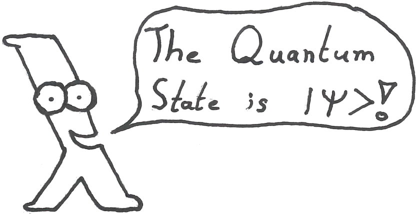

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  $f(\lambda)$ is the true quantum state of the system,
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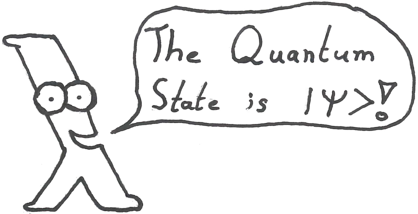

- There exists a function $f : \Lambda \to \mathcal{P}$
  $f(\lambda)$ is the true quantum state of the system,
  (bridge law as identity relation),
- Compatibility with quantum mechanical notion of quantum states:
  $\mu_\psi(f^{-1}(|\psi\rangle)) = 1.$

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2. If $\left|\psi\right\rangle \neq \left|\phi\right\rangle$, then μ_ψ and μ_ϕ are non-overlapping.

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- If $\mathcal{P}$ is the set of micro-canonical ensembles, then classical mechanics is P-ontic.
- If $\mathcal{P}$ also contains canonical ensembles, then classical mechanics is P-epistemic.

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Is just the energy ontic, or also the associated micro-canonical ensemble?

Are ψ-ontic Ontic Models ψ-ontic?

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Case study: The MKC hidden variable models.

Ontic Models and Hidden Variables

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The MKC models are constructed precisely to show the logical possibility of a **non-contextual hidden variable theory**. Allegedly, this possibility was ruled out by the Kochen-Specker theorem.

Assigning values to observables

For every non-degenerate self-adjoint operator A, there is a unique orthonormal basis

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For every non-degenerate self-adjoint operator A and every function f, the observables $f(A)$ and A can be measured jointly and the outcome for $f(A)$ is $f(a_j)$.

- The definite value assigned to $f(A)$ is determined by the definite value assigned to A.

Defending non-contextuality

Non-contextuality of quantum states

For two non-degenerate self-adjoint operators A, B, if

$$A\,|\psi\rangle = a\,|\psi\rangle \text{ and } B\,|\psi\rangle = b\,|\psi\rangle$$

then for every quantum state $|\phi\rangle$

$$\mathbb{P}_{|\phi\rangle}(A = a) = \mathbb{P}_{|\phi\rangle}(B = b) = |\langle\phi|\psi\rangle|^2\,.$$

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Non-contextuality of definite values

For two non-degenerate self-adjoint operators A, B, if

$$A\,|\psi\rangle = a\,|\psi\rangle \text{ and } B\,|\psi\rangle = b\,|\psi\rangle$$

then A has the value a iff B has the value b.

The Kochen-Specker Theorem

Assigning definite values

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For every ontic state: $|e\rangle$ is the “true” vector in an orthonormal basis $\mathcal{B}$ iff it is the true vector in every other orthonormal basis $\mathcal{B}'$ that contains $|e\rangle$.

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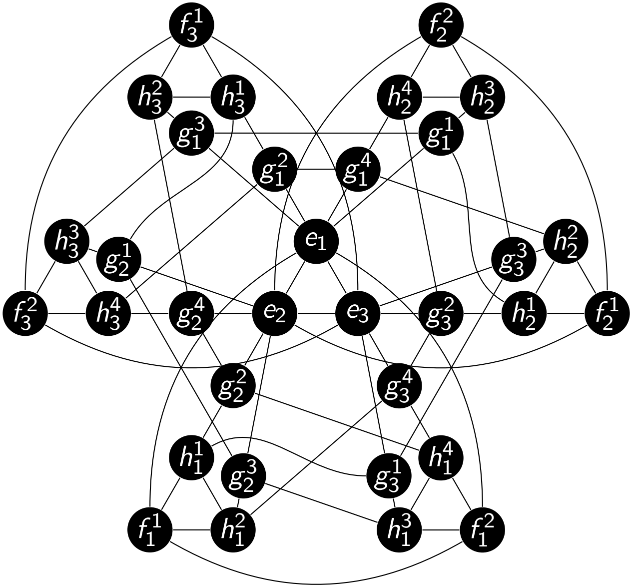
Kochen-Specker Theorem

There is a finite set of orthonormal bases for which one cannot select true vectors in a non-contextual way.

Uncolorable graph in 3 dimensions (Peres-cube)



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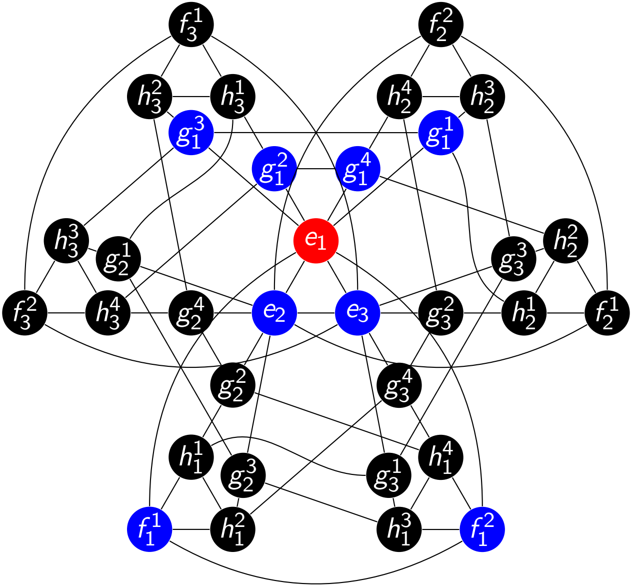

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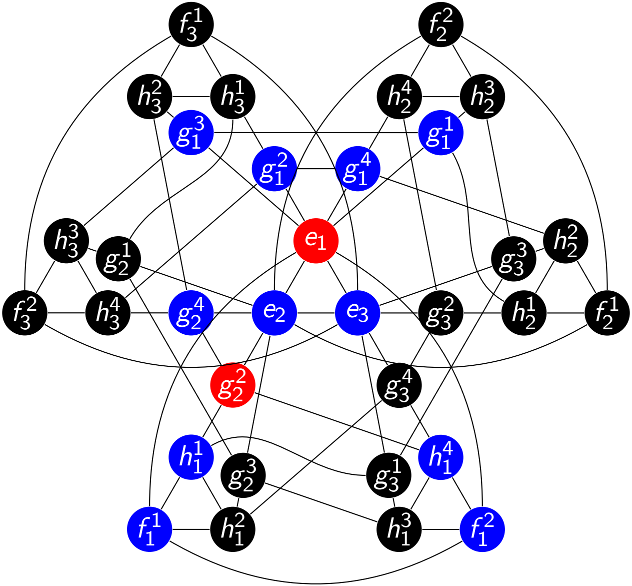

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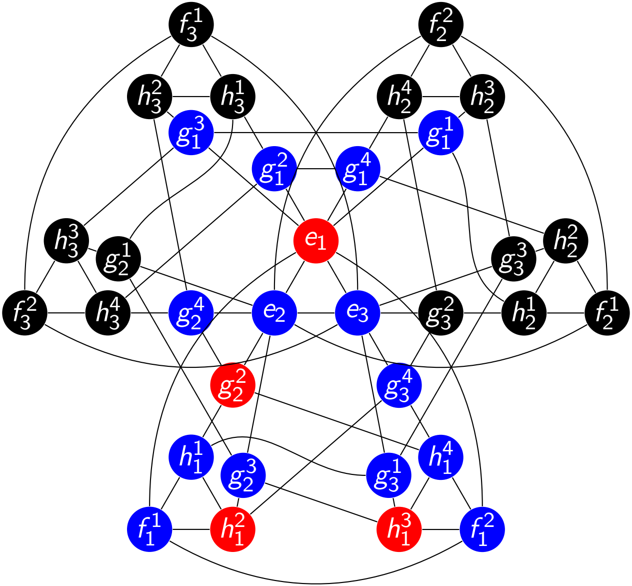

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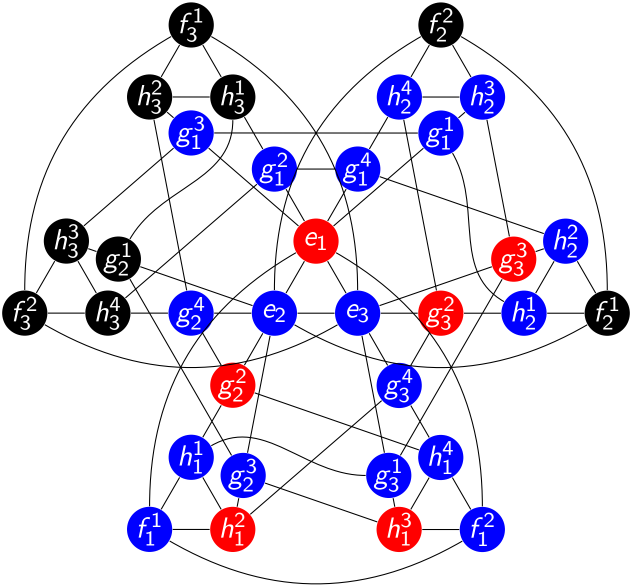

Circumventing the Kochen-Specker Theorem

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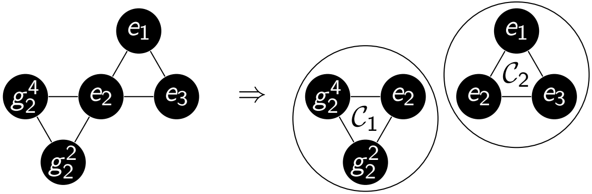
Bell: Value assigned to an observable depends on the context $\mathcal{C}$



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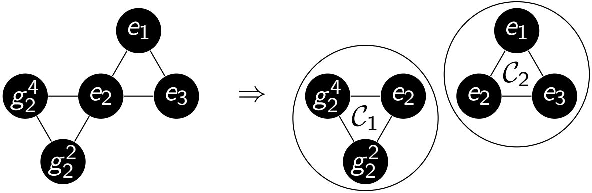
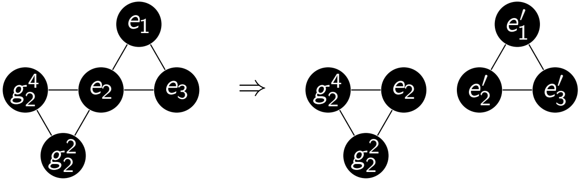


MKC: Not every orthonormal basis represents an observable



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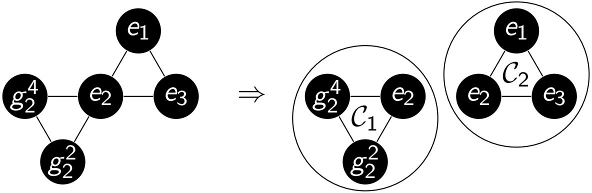

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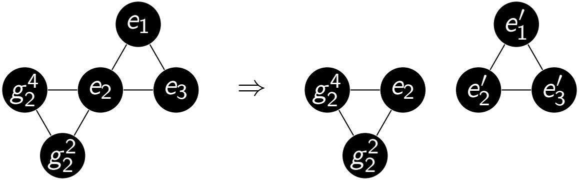

but every orthonormal basis can be approximated by an observable

$$0 < \|e_2 - e_2'\| < \epsilon.$$

The ontic models of Meyer, Kent and Clifton (MKC)

Definition

Two orthonormal bases

$$\mathcal{B}_1 = \left\{ \left| e_1^1 \right\rangle, \ldots, \left| e_n^1 \right\rangle \right\}, \; \mathcal{B}_2 = \left\{ \left| e_1^2 \right\rangle, \ldots, \left| e_n^2 \right\rangle \right\}$$

are *totally incompatible* if $\forall i, j = 1, \ldots, n$:

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Let $\mathfrak{B}$ be any set of pairwise totally incompatible orthonormal bases. The set of all self-adjoint operators with eigenvectors in one of the bases in $\mathfrak{B}$ is colorable.

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Non-Trivial Theorem (Clifton & Kent)

There exists a countable set $\mathfrak{B}$ of pairwise totally incompatible orthonormal bases that lies dense in the set of all orthonormal bases.

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- $|e_i^k\rangle \in \mathcal{B}_k$:

$$C_{e_i^k} := \{\lambda \in \Lambda \mid \lambda(k) = i\},$$
$$\mu_\psi(C_{e_i^k}) := |\langle\psi|e_i^k\rangle|^2.$$

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$$C_{e_i^k} := \{\lambda \in \Lambda \mid \lambda(k) = i\},$$
$$\mu_\psi(C_{e_i^k}) := |\langle\psi|e_i^k\rangle|^2.$$

- $|e_{i_1}^{k_1}\rangle \in \mathcal{B}_{k_1}$, $|e_{i_2}^{k_2}\rangle \in \mathcal{B}_{k_2}$, ..., $|e_{i_n}^{k_n}\rangle \in \mathcal{B}_{k_n}$:

$$C_{e_{i_1}^{k_1},\ldots,e_{i_n}^{k_n}} := \bigcap_{j=1}^{n} \{\lambda \in \Lambda \mid \lambda(k_j) = i_j\},$$
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BCLM Theorem

Any ontic model that reproduces the predictions of QM is not maximally ψ-epistemic (but "almost" ψ-ontic).

Are the MKC models ψ-ontic?

ψ-onticness:

For all pairs of quantum states ψ, ϕ, for all corresponding probability measures $\mu \in \Delta_\psi$, $\nu \in \Delta_\phi$ in the ontic model, the variational distance $D(\mu, \nu) := \sup_{\Omega \in \Sigma} |\mu(\Omega) - \nu(\Omega)|$ equals 1.

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Finding ontic states for $|\psi\rangle$

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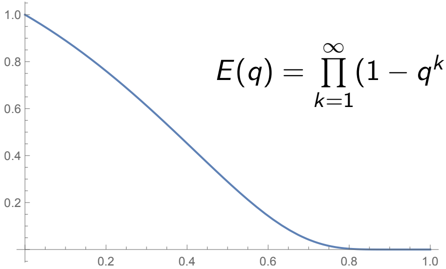
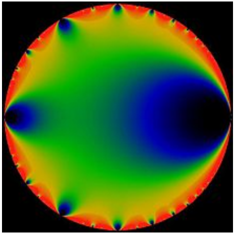


$$E(q) = \prod_{k=1}^{\infty}(1-q^k)$$



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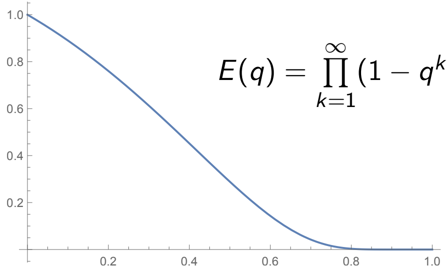

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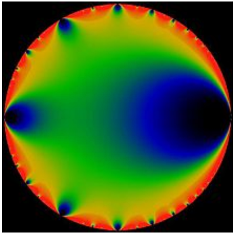

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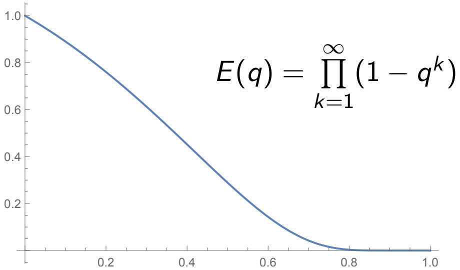

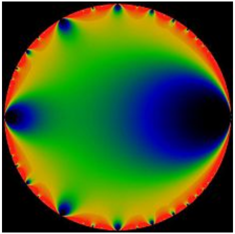

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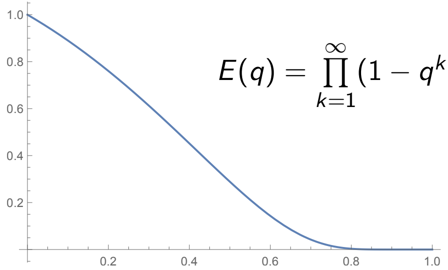

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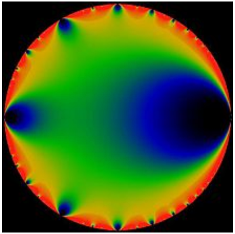

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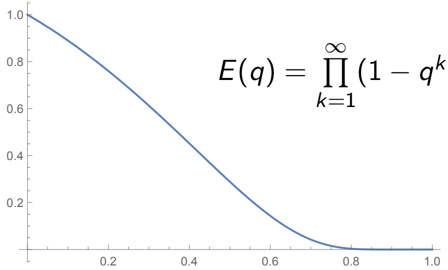

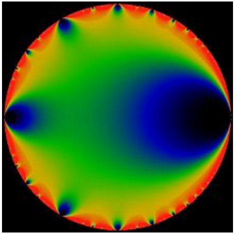

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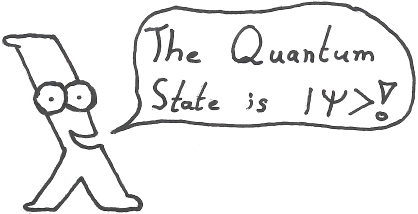
$\Rightarrow$ $D(\mu_\psi, \mu_\phi) = 1$

How ψ-ontic are the ψ-ontic MKC models?



1. There exists a function $f : \Lambda \to \mathcal{P}$, compatible with QM: $\mu_{|\psi\rangle}(f^{-1}(|\psi\rangle)) = 1$.
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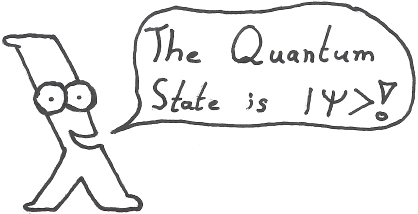

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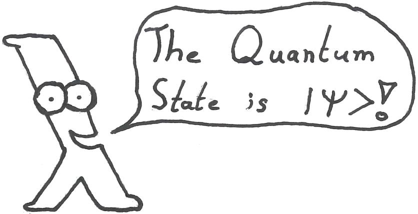

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- The MKC-models are ψ-ontic in the second sense: $D(\mu_\psi, \mu_\phi) = 1$.
- But we want the ontic state to tell us what $|\psi\rangle$ is, i.e., $\Lambda_\psi \subset \Lambda$ such that

$$\mu_\psi(\Lambda_\psi) = 1, \;\; \mu_\phi(\Lambda_\psi) = 0$$

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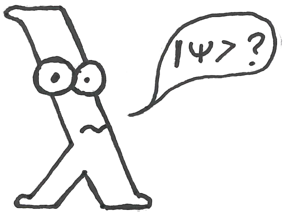

- $\Lambda_{\psi}^{\epsilon}$ does not contain all the ψ-ontic states: $\mu_{\psi}(\Lambda_{\psi}^{\epsilon}) < 1$.

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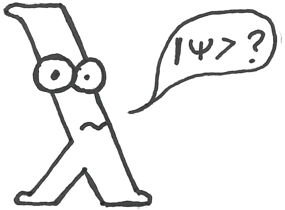

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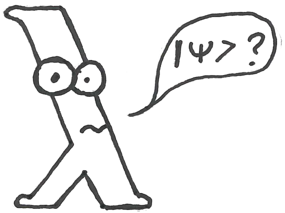

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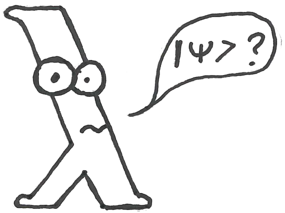

- Λ^ϵ_ψ does not contain all the ψ-ontic states: $\mu_\psi(\Lambda^\epsilon_\psi) < 1$.
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- Not all states in Λ^ϵ_ψ are ψ-ontic states:

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- The set Λ^ϵ_ψ is underdetermined by (ψ, ϵ).
- More generally: $f : \Lambda \to \mathcal{P}$ need not be unique, so we cannot speak of an identity relation.