ψ-ontic models without ψ

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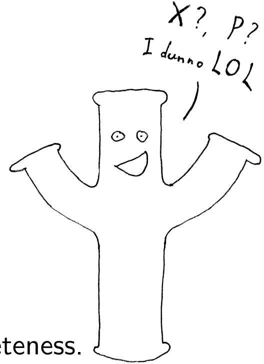
Where were you
When the PBR-paper came out?

¨The wavefunction is a real physical object after all, say researchers¨

Simpleminded counterexample?

- Traditional hidden variables complement ψ:

$$(|\psi\rangle, \lambda)$$



- Why hidden variables? Incompleteness!

$$\Delta X \Delta P \geq \frac{\hbar}{2}$$

- But $|\psi\rangle$ is a poor starting point for completeness.
- *Replace* $|\psi\rangle$ with λ.

  Contextuality, non-locality, etc.

Outline

1. Recap of the PBR Theorem
2. Highlighting the important/problematic step
3. The counterexample
4. Conclusion

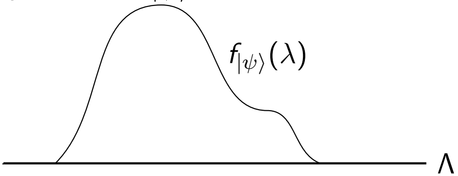
Statement of the Theorem

PBR Theorem

Any ontic model that reproduces the predictions of QM and satisfies the Preparation Independence Postulate is ψ-ontic.

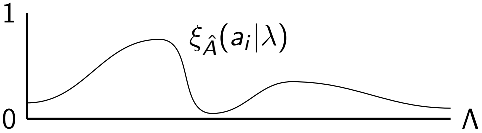
Ontic models for QM

- Preparation of $|\psi\rangle$:



$$\int f_{|\psi\rangle}(\lambda)\mathrm{d}\lambda = 1$$

- Measurement of $\hat{A}$:



$$\sum_i \xi_{\hat{A}}(a_i|\lambda) = 1$$

- Compatibility:

$$\int \xi_{\hat{A}}(a_i|\lambda) f_{|\psi\rangle}(\lambda)\, \mathrm{d}\lambda = |\langle\psi|a_i\rangle|^2$$

Statement of the Theorem

PBR Theorem

Any ontic model that reproduces the predictions of QM and satisfies the Preparation Independence Postulate is ψ-ontic.

> *We call a hidden variable model ψ-ontic if every complete physical state or ontic state in the theory is consistent with only one pure quantum state; we call it ψ-epistemic if there exist ontic states that are consistent with more than one pure quantum state. – Harrigan, Spekkens 2010*

$\Rightarrow$ The ontic state λ determines the quantum state $|\psi\rangle$.

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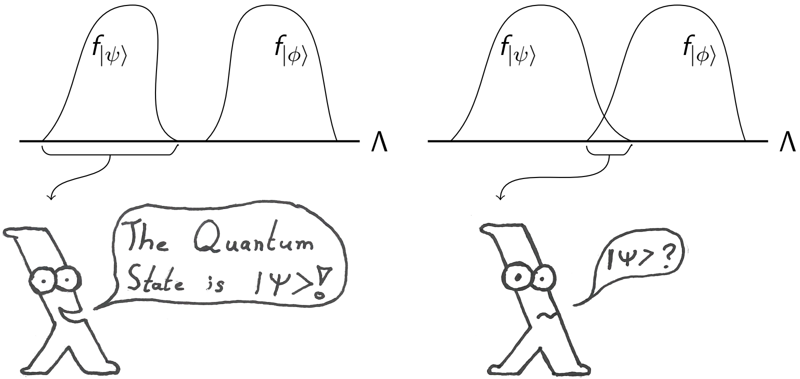
ψ-onticity

The ontic state λ determines the quantum state $|\psi\rangle$:

> *An important step towards the derivation of our result is the idea that the quantum state is physical if distinct quantum states correspond to non-overlapping distributions for λ. – Pusey, Barrett, Rudolph*



Four levels of ψ-onticity

1. There is a rule R assigning to every ontic state λ its corresponding quantum state $R(\lambda)$ such that
$$\mathbb{P}_{|\psi\rangle}(\{\lambda \mid R(\lambda) = |\psi\rangle\}) = 1.$$
2. For every $|\psi\rangle$ there is a set $\Lambda_{|\psi\rangle}$ such that for all $|\phi\rangle$:
$$\mathbb{P}_{|\psi\rangle}(\Lambda_{|\psi\rangle}) = 1 \text{ and } \mathbb{P}_{|\phi\rangle}(\Lambda_{|\psi\rangle}) = 0.$$
3. For every $|\psi\rangle$, $\epsilon > 0$ there is a set $\Lambda^{\epsilon}_{|\psi\rangle}$ such that for all $|\phi\rangle$:
$$\mathbb{P}_{|\psi\rangle}(\Lambda^{\epsilon}_{|\psi\rangle}) > 1 - \epsilon \text{ and } \mathbb{P}_{|\phi\rangle}(\Lambda^{\epsilon}_{|\psi\rangle}) < \epsilon.$$
4. For every $|\psi\rangle$ and $|\phi\rangle$ the distributions $\mathbb{P}_{|\psi\rangle}, \mathbb{P}_{|\phi\rangle}$ are non-overlapping.

Toy examples for QM show

$$3 \not\Longrightarrow 2 \not\Longrightarrow 1$$

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A contextual value definite ontic model

- A **context** C is a set of pairwise commuting 1-dimensional projectors that sum to $\hat{1}$.
- An **ontic state** λ is a function that assigns to each context a 1-dimensional projection operator such that

$$\lambda(C) \in C \ \forall C.$$

- The **value of an observable** $\hat{A}$ in the context C when the state is λ is

$$v_\lambda(\hat{A}|C) := \mathrm{Tr}\left(\lambda(C)\hat{A}\right).$$

- Satisfies
  - $v_\lambda(\hat{A}|C) \in \sigma(\hat{A})$,
  - $v_\lambda(f(\hat{A})|C) = f(v_\lambda(\hat{A}|C))$.

Recovering the Born rule

- Set of ontic states:

$$\Lambda = \{\lambda : \mathcal{C} \to L(\mathcal{H}) \mid \lambda(C) \in C\}.$$

- $C_1, C_2, \ldots, C_n$ contexts,
- $P_1, P_2, \ldots, P_n$ projectors such that $P_i \in C_i$, then

$$\Delta^{P_1,\ldots,P_n}_{C_1,\ldots,C_n} = \{\lambda \in \Lambda \mid \lambda(C_i) = P_i\}.$$

- For any $|\psi\rangle$:

$$\mathbb{P}_{|\psi\rangle}\left(\Delta^{P_1,\ldots,P_n}_{C_1,\ldots,C_n}\right) = \prod_{i=1}^{n} \langle\psi|P_i|\psi\rangle.$$

ψ-ontic without ψ

The model is ψ-ontic

For every $|\psi\rangle$ there is a set $\Lambda_{|\psi\rangle}$ such that for all $|\phi\rangle$:

$$\mathbb{P}_{|\psi\rangle}\left(\Lambda_{|\psi\rangle}\right) = 1 \text{ and } \mathbb{P}_{|\phi\rangle}\left(\Lambda_{|\psi\rangle}\right) = 0.$$

Proof:

- Choose countable sequence $C_1, C_2, \ldots$ such that

$$|\psi\rangle\langle\psi| \in C_i \ \forall i$$

- Define

$$\Lambda_{|\psi\rangle} = \{\lambda \in \Lambda \mid \lambda(C_i) = |\psi\rangle\langle\psi|\}$$

But without ψ

If $\langle\phi|\psi\rangle \neq 0$, then

$$\Lambda_{|\psi\rangle} \cap \Lambda_{|\phi\rangle} \neq \varnothing.$$

Conclusion

Negative conclusion:

- In the ontic model there are ontic states that are consistent with more than one pure quantum state. So, according to Harrigan & Spekkens, it is ψ-epistemic.

$\Rightarrow$ ψ-ontology theorems fails!

Positive conclusion:

> *I propose that we stop talking about the ill-defined notion of quantum state realism, and that we start talking instead about these sorts of question – e.g. whether quantum theory comes with objective standards for the ascription of states to physical situations. – Halvorson 2018*

- The fact that quantum states correspond to non-overlapping distributions indicates that two agents using distinct quantum states are substantially disagreeing about *something*.